**The Four Aristotelian Forms** ∀⟺ ∃*→*

* *All P’s are Q’s.* **∀x P(x)** → **Q(x)**
* *Some P’s are Q’s.* **∃x P(x) & Q(x)**
* *No P’s are Q’s.* **~(∃x P(x) & Q(x))**
  + **alternatively, ∀x P(x)** → **~Q(x)**
* *Some P’s are not Q’s.* **∃x (P(x) & ~Q(x))**

**Existential Noun Phrases**

* Some dog has the property Q.  
  ∃x Dog(x) & Q(x)
* A cube has the property Q.  
  Something that is a Cube has the property Q  
  ∃x Cube(x) & Q(x)

**Universal Noun Phrases**

* Every/Each dog has a home.  
  All dogs have homes.  
  ∀x Dog(x) *→* HasHome(x)

**When is a quantified sentence a tautology?**

When its truth-functional form is a tautology.

**When is an argument containing quantified sentences tautologically valid?**

When its truth functional form is tautologically valid.

**What quantified sentences are logical truths?**

Before we considered quantified sentences, we checked for logical truths by using a truth table. By considering solely the connectives and the truth values of all the atomic sentences of a complex sentence, the latter was a logical truth if the main connective was always true for all logically possible circumstances, and a tautology if it was true for all circumstances.

When analyzing quantified sentences, we need to incorporate the quantifiers and identity predicate into our analysis. If a sentence is logically true when we take these into account (leaving out the names and meanings of the predicates), then a sentence is a first-order validity, ie a sentence that is always true.

Recall that a quantified sentence being true is determined by the relationships between collections of objects. In the simplest cases, we have two cases: i) a quantified sentence with an existential quantifier ∃x S(x) which is true if and only if there is at least one object in the domain of discourse that satisfies the wff S(x), ie at least one object x such that x has the property x; ii) a quantified sentence with a universal quantifier ∀x S(x) which is true if and only if all objects in the domain of discourse satisfy the wff S(x), ie all objects have the property S.

Therefore, if a quantified sentence is logically true then it is always true in a particular world: the

There is no way, at this stage, to prove that a quantified sentence is a logical truth. We can prove that a quantified sentence is not a logical truth by finding a counterexample, ie a circumstance in which the quantified sentence is false.

**What arguments involving quantification are valid?**

If we analyze an argument taking into account not only connectives and atomic sentences, but also quantifiers an quantified atomic sentences, and conclude that the argument is valid, then the conclusion is a first-order consequence of the premises.

**is there a way to conclude that an argument or a sentence is FO valid?**

There is no precise and mechanical procedure such as we had with truth tables in the case of analyzing validity of sentences and arguments not involving quantifiers. We are able to prove that something is not FO valid, by finding a counterexample

**Some relationships**

* If S is a tautology, then it is FO valid, and also a logical truth.
* If S is FO valid, then it is a logical truth.
* There are many FO validities that are not tautologies, and many logical truths that are not FO validities.
* FO validity takes into account FO structure, which includes quantifiers, the identity predicate, and connectives.
* Logical validity takes into account the meanings and names of predicates as well.
* Tautological validity only takes into account the connective structure.

**What are valid patterns of inference involving quantifiers?**

Before establishing patterns of inference for quantified sentences, it is important to show one result before:

two wffs with free variables are logically equivalent if in any possible circumstance they are satisfied by the same objects. This means that if we apply the same quantifiers to logically equivalent wffs the resulting quantified sentences are logically equivalent.

More generally, let P and Q be wffs, possibly containing free variables, and let S(P) be any sentence containing P as a component part. Then if P and Q are logically equivalent: P ⟺ Q, then so too are S(P) and S(Q): S(P) ⟺ S(Q). Ie we can interchange logically equivalent wffs in a quantified sentence to obtain a logically equivalent quantified sentence.

Now to the equivalences.

There are equivalences involving **DeMorgan laws for quantified sentences**. With non-quantified DeMorgan laws show how negation relates to & and | connectives in a sentence. In quantified sentences, DeMorgan laws show us how negation interacts with the quantifiers.

The DeMorgan laws for quantifiers are aka **quantifier/negation equivalences**

**~∀x P(x)** ⟺ **∃x ~P(x)**

**~∃x P(x)** ⟺ **∀x ~P(x)**

We basically push a negation sign past the quantifier by flipping the quantifier to the other quantifier.

Recall the Aristotelian forms:

* *All P’s are Q’s.* **∀x P(x)** → **Q(x)**
* *Some P’s are Q’s.* **∃x P(x) & Q(x)**
* *No P’s are Q’s.* **~(∃x P(x) & Q(x))**
  + **alternatively, ∀x P(x)** → **~Q(x)**
* *Some P’s are not Q’s.* **∃x (P(x) & ~Q(x))**

We can apply DeMorgan laws to these quantified sentences and see that they are related through negation.

* ***All P’s are Q’s.* ∀x P(x) → Q(x)**
  + **when negated becomes**
    - ~(∀x P(x) → Q(x))  
      **∃**x ~(P(x) → Q(x))  
      ∃x ~(~P(x) | Q(x))  
      ∃x P(x) & ~Q(x))  
      **“Some P’s are not Q’s”**
* ***Some P’s are Q’s.* ∃x P(x) & Q(x)**
  + **when negated becomes**
    - ~(∃x P(x) & Q(x))  
      ∀x ~(P(x) & Q(x))  
      ∀x (~P(x) | ~Q(x))  
      ∀x (P(x) → ~Q(x)  
      “No P’s are Q’s”
* ***No P’s are Q’s.* ~∃x P(x) & Q(x)**
  + when negated becomes
    - ∃x P(x) & Q(x)  
      Some P’s are Q’s
* ***Some P’s are not Q’s.* ∃x (P(x) & ~Q(x))**
  + **when negated becomes**
    - ~∃x (P(x) & ~Q(x))  
      ∀x ~(P(x) & ~Q(x))  
      ∀x (~P(x) | Q(x))  
      ∀x (P(x) → Q(x))  
      All P’s are Q’s.

Next we have some equivalences involving & and |

∀x (P(x) & Q(x)) ⟺ ∀x P(x) & ∀x Q(x)

∀x (P(x) | Q(x)) not logically equivalent to ∀x P(x) | ∀x Q(x)

∃x (P(x) | Q(x)) ⟺ ∃x P(x) | ∃x Q(x)

∃x (P(x) & Q(x)) not logically equivalent to ∃x P(x) & ∃Q(x)

**Note that in general** we can pass the universal quantifier into a conjunction, and we can pass an existential quantifier into a disjunction, but we can’t pass a universal quantifier into a conjunction nor can we pass an existential quantifier into a conjunction.

There is an exception to these observations, which occurs when the variable being quantified does not appear in one of the terms in the conjunction or disjunction.

For example, ∀x (P | Q(x)) ⟺ P | ∀x Q(x).

Previously we could not pass the universal quantifier into the disjunction, but now because P does not contain the variable x, we can.

“For every x, either the property P is true or x has the property Q.” is equivalent to “Either property P is true or every x has the property Q.

Similarly, ∃x (P & Q(x)) ⟺ P & ∃x Q(x).

That is we can pass the existential quantifier into a conjunction when one of the conjuncts does not contain the quantified variable.

“For some x, P is true and x has the property Q” equivalent to “P is true and for some x, x has the property Q”

**What is the axiomatic method?**

We have a world and we wish to infer conclusions about a part of this world.

We write sentences involving predicates that are specific to concepts or relationships present in this specific part or domain of the world.

We write arguments that are logically valid when we take into account these predicates, but not necessarily logically valid when we consider only the first order structure (connectives, quantifiers, identity predicate).

If, in such an argument, we add premises that make explicit the meanings and relationships between and among predicates, we can make an argument that wasn’t first-order valid into an argument that is first-order valid.

This technique of adding premises whose truth value is justified by the meanings of the predicates is one aspect of the axiomatic method. Such premises are called meaning postulates.

**What do we know about sentences with multiple quantifiers?**

* the order of the quantifiers usually makes a difference
* the quantifiers don’t range over distinct objects (the domain of all objects is the same for all quantifiers)
* certain English sentences are ambiguous in the sense that we can interpret them in more than one way, and therefore translate them to FOL in more than one way
  + sometimes this means that we can read the sentence in two ways, one of which implies the other way; the one that implies the other is called the strong reading, and the implied is called the weak reading
* prenex form is a form that a quantified sentence can be in if all the quantifiers are out in front before the part of the sentence that contains no quantifiers
  + every sentence can be put into a logically equivalent prenex formSim